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Isospin Symmetry Breaking Effects on the Properties of Asymmetrical Nuclear Matter and β-Stable Matter

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We have studied the influences of isospin symmetry breaking of nucleon–nucleon interaction on the various properties of asymmetrical nuclear matter and β-stable matter. For asymmetrical nuclear matter, it is found that by including this isospin symmetry breaking, the changes of bulk properties increase by increasing both density and asymmetry parameter. However, these effects on the total energy and equation of state of β-stable matter are ignorable. For asymmetrical nuclear matter, the validity of the empirical parabolic law in the isospin symmetry breaking case is shown. It is observed that the isospin symmetry breaking of nucleon–nucleon interaction affects the β-equilibrium conditions in β-stable matter.

KEY WORDS: nucleon–nucleon interaction; isospin symmetry breaking; nuclear matter; β-stable matter.

1. INTRODUCTION

In the calculation of the properties of nucleonic matter, the nucleon–nucleon potential has a crucial role. Recent models of two-nucleon potentials which fit the nucleon–nucleon scattering data with high precision, contain terms which break the isospin symmetry (Li and Machleidt, 1998; Machleidt and Slaus, 2001; Stoks *et al.*, 1994). Isospin symmetry is the invariance under any rotation in the isospin space. But, because of the mass difference between neutral pion and charged pion ($\Delta m \simeq 4.6$ MeV), this symmetry is broken (Li and Machleidt, 1998; Machleidt and Slaus, 2001; Stoks *et al.*, 1994). This means that for the new two-nucleon potentials, the neutral pion exchange is distinguished from charged pion exchange. This distiction implies that the neutron–neutron (nn), neutron–proton (np), and proton–proton (pp) interactions are different and therefore, these

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potentials are called isospin symmetry breaking (ISB) potentials (Stoks *et al.*, 1994).

Because of the distinction between the neutral pion and charged pion exchange, for the potentials which describe the long-range part of the nucleon–nucleon interaction in terms of the one-pion exchange model, the one-pion exchange term is one of the origins of ISB (Li and Machleidt, 1998; Machleidt and Slaus, 2001; Stoks *et al.*, 1994). For the ISB potentials, in the isospin $T = 1$ states, the distinction between the nn, np, and pp potentials is necessary (Stoks *et al.*, 1994). Therefore, in the calculations with these potentials, the difference between the nn, np, and pp wave functions should be considered. In our calculations, we use the Reid-93 potential (Stoks *et al.*, 1994). This potential describes the longe-range part of the nucleon–nucleon interaction in terms of the one-pion exchange model accounting for the mass difference between the neutral pion π^0 and charged pions π^+ and π^-. In the Reid-93 potential, for the non-one-pion exchange parts, additional ISB terms have been included in the $^1S_0$ state potential (Stoks *et al.*, 1994).

In this work, we intent to calculate the various properties of asymmetrical nuclear matter and β-stable matter in two different cases with isospin symmetry (IS) and with isospin symmetry breaking (ISB). By comparing the results of the IS and ISB cases, we study the influence of ISB of nucleon–nucleon interaction on these properties. For the ISB case, we use the Reid-93 potential (Stoks *et al.*, 1994) in which the one-pion exchange potentials for the np and pp (nn) interactions as well as the np and pp (nn) $^1S_0$ potentials are different. However, for the IS case, we replace the one-pion exchange and $^1\mathrm{S}_0$ potentials of nn and pp interactions with the corresponding np interaction (Stoks *et al.*, 1994). In our calculations, we employ the formalism used in our previous works (Bordbar, 2002a,b; Bordbar and Modarres, 1997, 1998; Bordbar and Riazi, 2001, 2002; Modarres and Bordbar, 1998) in which we consider the distiction between the partial waves with $M_T = +1$, 0, and -1. This method is a variational approach based on the cluster expansion of the energy functional containing the state-dependent correlation functions. The convergence of the results of this approach has been also tested by calculating the three-body cluster term (Bordbar and Modarres, 1997).

2. ISOSPIN SYMMETRY BREAKING EFFECTS ON THE ASYMMETRICAL NUCLEAR MATTER PROPERTIES

The asymmetrical nuclear matter properties are important for studying the supernova collapse and heavy-ion reaction (Lattimer and Prakash, 2000; Li *et al.*, 1997). For asymmetrical nuclear matter, we define the asymmetry parameter β as

$$\beta = \frac{\rho_\mathrm{n} - \rho_\mathrm{p}}{\rho}, \tag{1}$$

where ρ_n and ρ_p are the number densities of neutrons and protons, respectively, and $\rho = \rho_n + \rho_p$ is the total number density. This implies that for symmetrical nuclear matter ($\rho_n = \rho_p$) $\beta = 0.0$ and for the pure neutron matter $\beta = 1.0$.

In this section, we present our results for the various properties of asymmetrical nuclear matter and then we investigate the effects of ISB on these properties.

The total energy of asymmetrical nuclear matter as a function of total number density for various asymmetry parameters is given in Fig. 1. It is seen that for both IS and ISB cases as the asymmetry parameter increases, the saturation point shifts to the lower densities. However, for the large values of β, the energy curve does not shows any minimum. The changes of total energy due to the inclusion of ISB for nucleon–nucleon interaction are shown in Fig. 2. This figure shows that

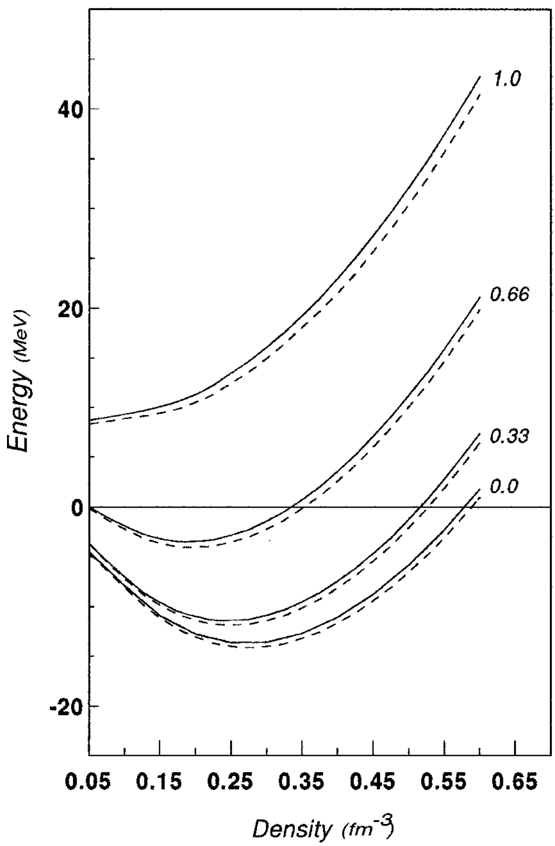

Fig. 1. The total energy of asymmetrical nuclear matter versus density for $\beta = 0.0, 0.33, 0.66$, and 1.0 in the IS (dashed curves) and ISB (full curves) cases.

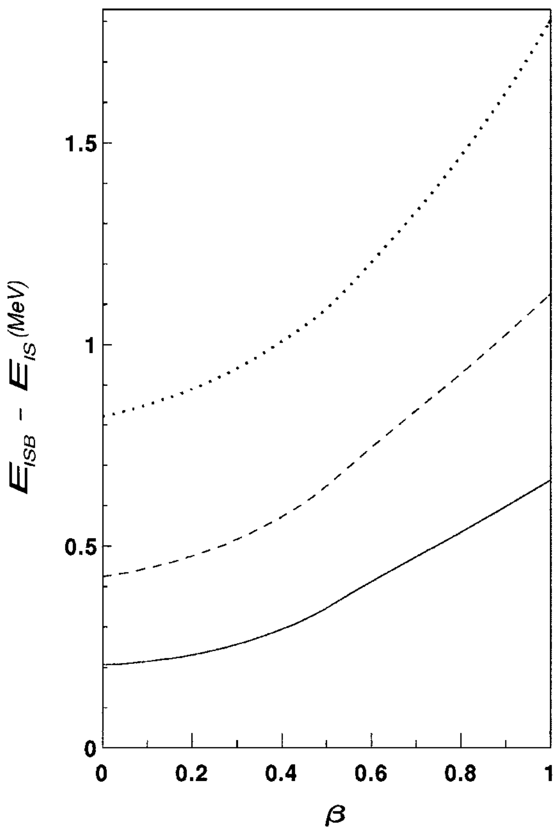

Fig. 2. The difference between total energy of ISB case (E_{ISB}) and total energy of IS case (E_{IS}) versus asymmetry parameter β for densities $\rho = 0.15$ (full curve), 0.3 (dashed curve), 0.6 (dotted curve) fm^{-3}.

the differences between our results in the ISB and IS cases increase by increasing both density and asymmetry parameter. In Table I, our results for the differences between the ISB and IS energies at $\beta = 0.0$ are compared with those of MPM (Muther *et al.*, 1999). It can be seen that our results are in a good agreement with the results of MPM for the Nijm-II potential.

The β-dependence of the total energy of asymmetrical nuclear matter in the ISB case is shown in Fig. 3 for different values of density. It can be seen that even in the presence of ISB, the asymmetrical nuclear matter energy is nearly linear in β^2 and therefore, the higher orders effects of the asymmetry are very small. This confirms the validity of the empirical parabolic law.

Table I. The Comparison Between Our Results for the Energy Differences of ISB and IS Cases at $\beta = 0.0$ and Those of MPM (Muther *et al.*, 1999)

	$E_{\text{ISB}} - E_{\text{IS}}$ (MeV)					
Density (fm^{-3})	Our results	MPM (Reid-93)	MPM (Nijm-I)	MPM (Nijm-II)	MPM (CD Bonn)	MPM (AV_{18})
0.10	0.14	0.17	0.22	0.16	0.22	0.21
0.15	0.21	0.21	0.28	0.18	0.28	0.25
0.25	0.34	0.25	0.51	0.32	0.51	0.41
0.35	0.48	0.28	0.78	0.43	0.79	0.55
0.40	0.56	0.29	0.87	0.51	0.89	0.61

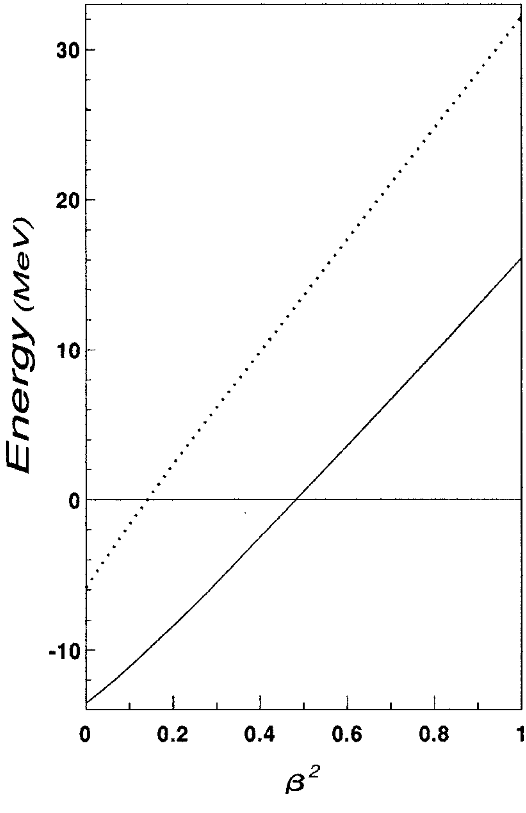

Fig. 3. The total energy of asymmetrical nuclear matter in the ISB case versus β^2 for $\rho = 0.3$ (full curve) and 0.5 (dotted curve) fm^{-3}.

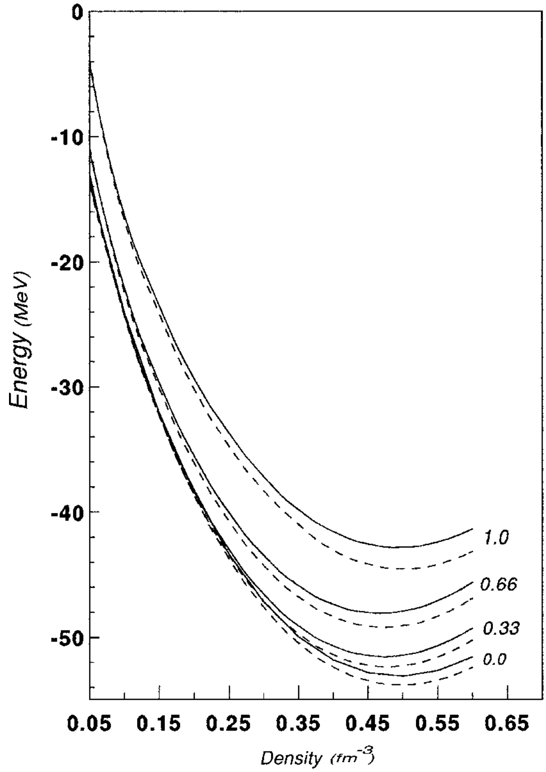

Fig. 4. As Fig. 1, but for the potential energy of asymmetrical nuclear matter.

In Fig. 4, we have shown the effects of ISB on the potential energy of asymmetrical nuclear matter. It is seen that the potential energy difference between the IS and ISB cases increases by increasing asymmetry parameter β. This is due to the fact that by increasing β, the energy contribution of nn interaction increases. Therefore, the difference between the case in which we distinguish between the nn and np interactions (ISB case) and the case in which we replace the nn interaction by the corresponding np interaction (IS case) increases by increasing β. It is also seen that for each value of density and asymmetry parameter, the potential energy shift due to the ISB is positive. This behavior is reflecting the fact that in the partial wave with isospin $T = 1$, the $M_T = 0$ potential is more attractive than the corresponding $M_T = +1$ and $M_T = -1$ potentials (Bordbar, 2003).

The influence of ISB on the saturation point properties of asymmetrical nuclear matter is shown in Table II. We see that the ISB effects on the saturation density, energy, and incompressibility of asymmetrical nuclear matter are generally small, but they are considerable for large β.

Table II. The Effects of ISB of Nucleon–nucleon Interaction on the Saturation Density (fm^{-3}), Energy (MeV), and Incompressibility (MeV) of Asymmetrical Nuclear Matter for Different Values of Asymmetry Parameter β

	ISB			IS		
β	Density	Energy	Incompressibility	Density	Energy	Incompressibility
0.0	0.272	−13.81	224	0.277	−14.08	233
0.33	0.245	−11.39	196	0.252	−11.85	206
0.66	0.185	−3.47	112	0.196	−4.02	124

The nuclear matter asymmetry energy is of particular interest in astrophysics, especially for studying the supernova mechanism (Lattimer and Prakash, 2000). The asymmetry energy can be approximated as the difference between binding energies of pure neutron matter ($\beta = 1.0$) and symmetrical nuclear matter

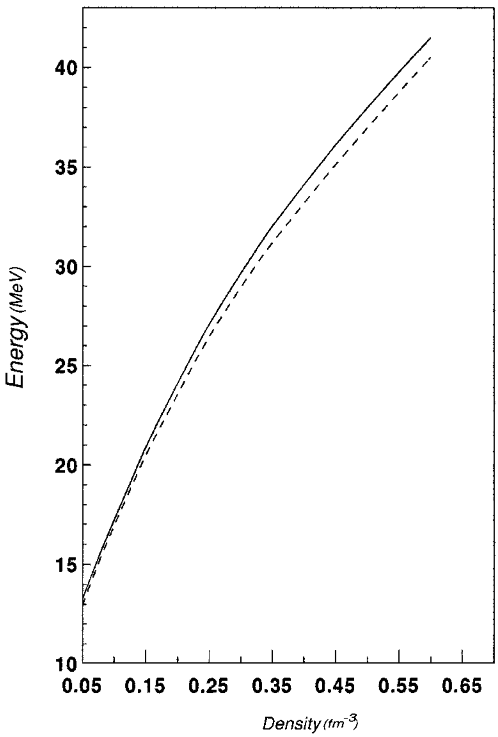

Fig. 5. The asymmetry energy as a function of density for the IS (dashed curve) and ISB (full curve) cases.

($\beta = 0.0$). The validity of using this approximation (called parabolic approximation) in calculating the asymmetry energy was shown in Fig. 3. Our results for the asymmetry energy calculated for both IS and ISB cases are presented in Fig. 5. It can be seen that the changes of asymmetry energy due to the inclusion of ISB increase by inceasing density.

3. ISOSPIN SYMMETRY BREAKING EFFECTS ON THE β-STABLE MATTER PROPERTIES

The properties of β-stable matter are of special importance in astrophysics. The equation of state of β-stable matter is very important for investigating the neutron stars, particularly in their stability (Akmal *et al.*, 1998; Heiselberg and

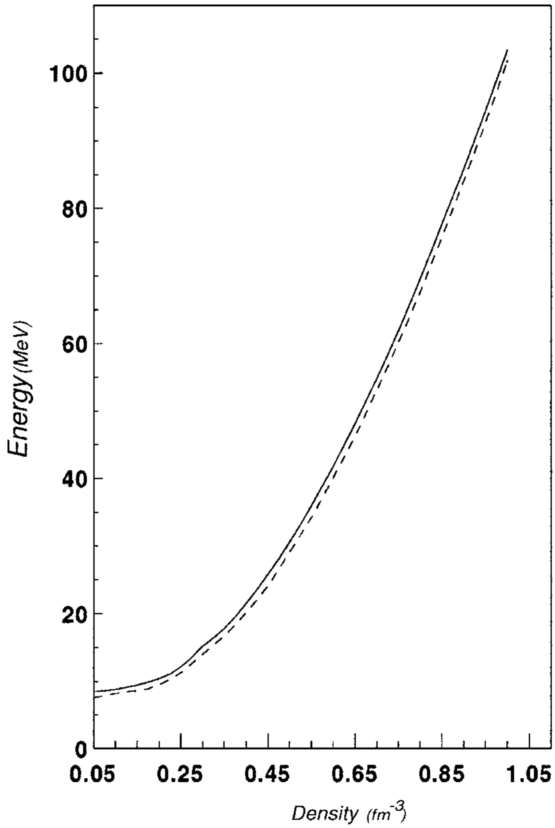

Fig. 6. The total energy of β-stable matter as a function of density for the IS (dashed curve) and ISB (full curve) cases.

Pandharipande, 2000). The β-stable matter is a composition of nucleons and leptons which is electrically neutral and in β-equilibrium. The β-equilibrium means that there is a balance between the competing processes of electron capture $e + p \rightarrow n + \nu$ and β-decay $n \rightarrow e + p + \bar{\nu}$. This equilibrium conditions determine the proton fraction ($x = \rho_p/\rho$) in β-stable matter. The proton fraction is important for studying the cooling of a neutron star (Prakash, 1994).

In Fig. 6, we have plotted our results for the total energy of β-stable matter versus density in the IS and ISB cases. It is seen that by considering ISB for the nucleon–nucleon interaction, the change of β-stable matter energy is small. For example, at density $\rho = 1.0\ \mathrm{fm}^{-3}$, the energy correction due to the ISB effects is about 1.6%. Therefore, the effects of ISB on the total energy of β-stable matter are negligible.

The calculated energy contribution of leptons and proton fraction in β-stable matter are shown in Figs. 7 and 8. It is found that these properties of β-stable matter

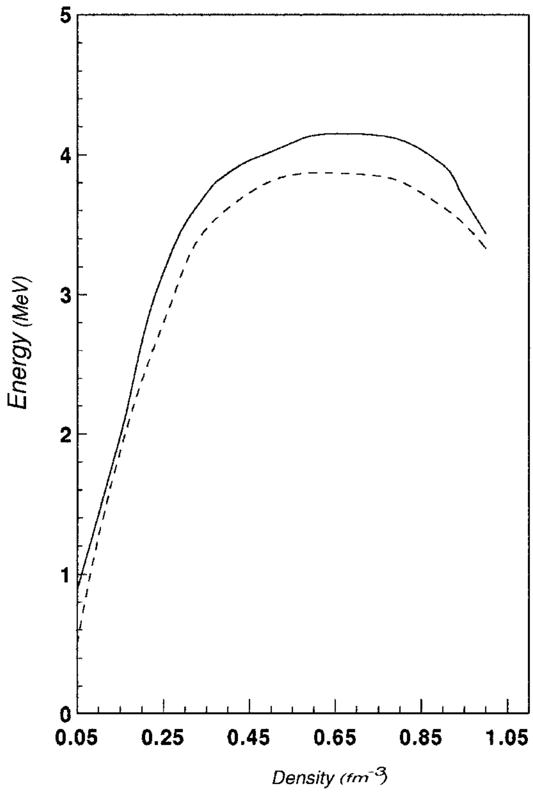

Fig. 7. As Fig. 6, but for the energy contribution of leptons.

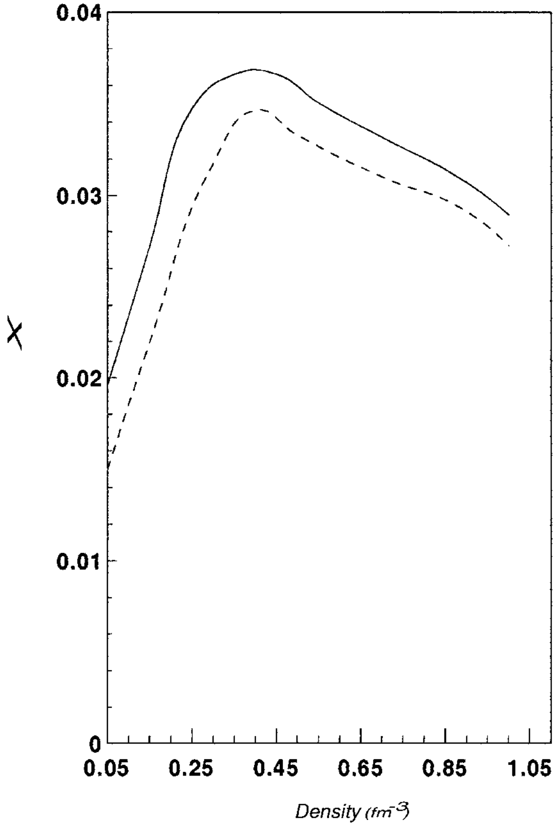

Fig. 8. As Fig. 6, but for the proton fraction.

are varied by the inclusion of ISB of nucleon–nucleon interaction. This shows that the ISB of nucleon–nucleon interaction affects the conditions of β-equilibrium.

The equation of state of β-stable matter calculated for both IS and ISB cases are shown in Fig. 9. It can be seen that the influences of ISB on the equation of state of β-stable matter are ignorable.

4. SUMMARY AND CONCLUSION

The two-nucleon potential has a crucial role in the nuclear many-body calculations. By comparing the results of two different cases of isospin symmetry (IS) and isospin symmetry breaking (ISB) for the nucleon–nucleon interaction, we have evaluated the effects of ISB on the different properties of asymmetrical nuclear matter and β-stable matter. It was seen that the changes of total energy,

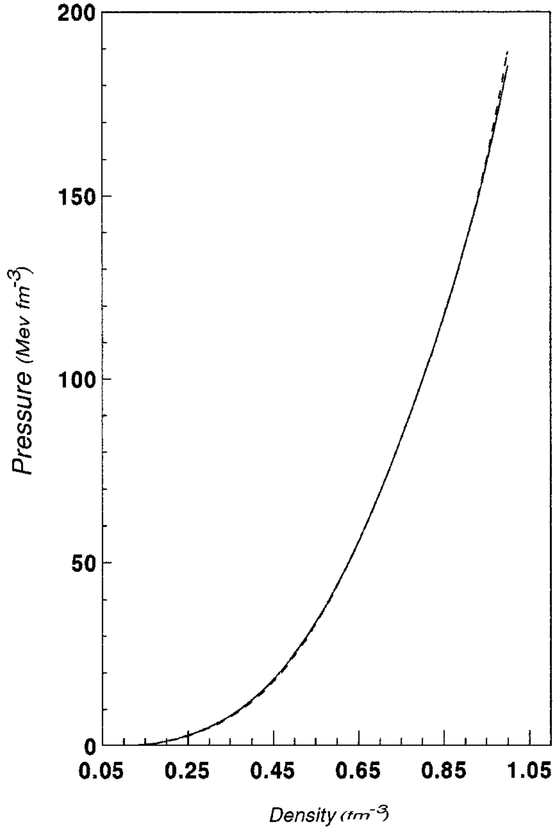

Fig. 9. As Fig. 6, but for the equation of state of β-stable matter.

potential energy, and asymmetry energy of asymmetrical nuclear matter due to the inclusion of ISB for the nucleon–nucleon interaction increase by increasing both density and asymmetry parameter. It was shown that the empirical parabolic law is valid, even by considering ISB for the nucleon–nucleon interaction. It was indicated that our results for the total energy and the equation of state of β-stable matter in the IS and ISB cases are nearly identical. For β-stable matter, it was shown that the ISB affects the energy contribution of leptons and proton fraction.

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